Comments on "Critical Study on the Absorbing Phase Transition in a Four-State Predator-Prey Model in One Dimension"

P. K. Mohanty, Rakesh Chatterjee and Abhik Basu

Theoretical Condensed Matter Physics Division,
Saha Institute of Nuclear Physics, Kolkata 700064, India.

Abstract.

In a recent article [arXiv:1108.5127] Park has shown that the four-state predator-prey model studied earlier in *J. Stat. Mech, L05001 (2011)* belongs to Directed Percolation (DP) universality class. It was claimed that predator density is not a reasonable order parameter, as there are many absorbing states; a suitably chosen order parameter shows DP critical behaviour. In this article, we argue that the configuration that does not have any predator is the only dynamically accessible absorbing configuration, and the predator density too settles to DP critical exponents after a long transient.

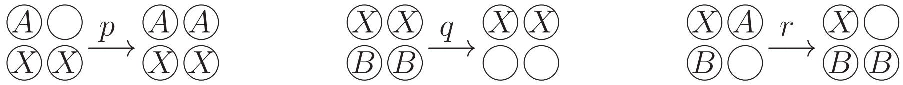
PACS numbers: 64.60.ah, 64.60.-i, 64.60.De, 89.75.-k

Systems having absorbing configurations may undergo a non-equilibrium phase transition [1] from an active to an absorbing state. The critical behavior of these absorbing state phase transitions (APTs) [2] depends on the the symmetry of the order parameter and presence of additional conservation laws. It has been conjectured [3] that in absence of any special symmetry the APT belongs to the directed percolation (DP) universality class as long as the system has a single absorbing state.

Since the coarse grained microscopic theory of DP, which is a birth-death-diffusion process, is based on a single component Reggeon field theory[4], critical behavior in presence of additional field is expected to alter the critical behavior. The additional field may bring in multiple absorbing states and/or additional conservation laws. Presence of multiple absorbing states may [5] or may not [6] affect the universality. Coupling of order parameter to a conserved field too lead to DP [7] or non-DP [8] critical behavior. The models of directed percolation with more than one species [9], which brings in additional coarse grained fields, has also been studied [10]. The predator-prey cellular automaton models [11] in higher dimension too shows an APT to an absorbing (extinct) state which belongs to DP-class. The role of additional fields in these models are not quite well understood.

Recently we studied a predator-prey model[12] on a $(1+1)$-dimensional lattice, where each lattice site is either vacant, occupied by a predator A, a prey B or both (one A and one B). In these four state predator-prey (4SPP) model growth of preys and death of predators occurs independently, whereas death of a prey is always followed by instant birth of a predator. Based on the numerical simulations and estimated critical exponents, we have suggested the possibility of a new universality class. In particular, the decay of clusters at the critical point was found to be distinctly different from those of DP. However, in a recent article Park [13] has suggested a different scenario. It was claimed that the predator density ρ_B can not be taken as a order parameter as there are infinitely many absorbing states. The transition is found to be in DP class, when order parameter is chosen suitably. In this article, we show that although there are many absorbing states, only one of them ($\rho_A = 1, \rho_B = 0$) is dynamically accessible. In fact the order parameter ρ_B which was showing an apparently new critical behavior, slowly crosses over to DP.

For completeness, first let us define the model. On a one dimensional periodic lattice, each site is either vacant, or occupied by a single particle A (prey), or occupied by a single particle B (predator) or by both particles (co-existing A and B); correspondingly at each site i we have $s_i = 0, 1, 2, 3$. Alternatively one may describe the 4-state predator-prey (4SPP) model considering two separate branches, one for A and the other for B particles, where particles living in one branch can not move to the other. We consider only the asymmetric case of the model which evolves according to the following dynamics, where X denotes both presence and absence of particles in respective branches.



Although, it was not explicitly mentioned in our earlier report [12], numerical simulations of the model was carried out in using exactly the same state variables $b_i = 4s_i + s_{i+1}$ mentioned in [13]. Clearly these bond variables $b_i = 0, 1 \ldots 15$ follow a dynamical rules,

$$4 \xrightarrow{p} 5; 6 \xrightarrow{p} 7; 12 \xrightarrow{p} 13; 14 \xrightarrow{p} 15; \quad 9 \xrightarrow{r} 10; 13 \xrightarrow{r} 14 \tag{1}$$

$$10 \xrightarrow{q} 0; 11 \xrightarrow{q} 1; 14 \xrightarrow{q} 4; 15 \xrightarrow{q} 5; \tag{2}$$

The neighbors are updated along with b_i as follows. For dynamics (1) b_{i+1} is increased by 4 whereas for dynamics (2) b_{i+1} and b_{i-1} are decreased by 8 and 2 respectively. Clearly a bond i is active when b_i is either 4 or 6, or it is greater than 8. It was argued by Park [13] that the density of active bonds ρ, the correct order parameter (as there are infinitely many absorbing states) of the system, vanishes at the critical point $p_c = 0.15381$ when $q = 0.02$ and $r = 0.9$. For the same parameter values, we had estimated earlier that predator density ρ_B vanishes at $p_c^B = 0.1484$. This raises a question that possibly in the region $p_c^B < p \leq p_c$, the system falls into an absorbing state which has isolated Bs. However a configuration with one or more isolated B can be absorbing only when there are no As. Thus, at p_c we must have $\rho_A = 0$. It was evident from Fig. 2(a) of Ref. [12] that $\rho_A > 0 \;\; \forall p$. This indicates that ρ_B must vanish at the same value of p, possibly at $p_c = 0.15381$, as estimated by Park [13].

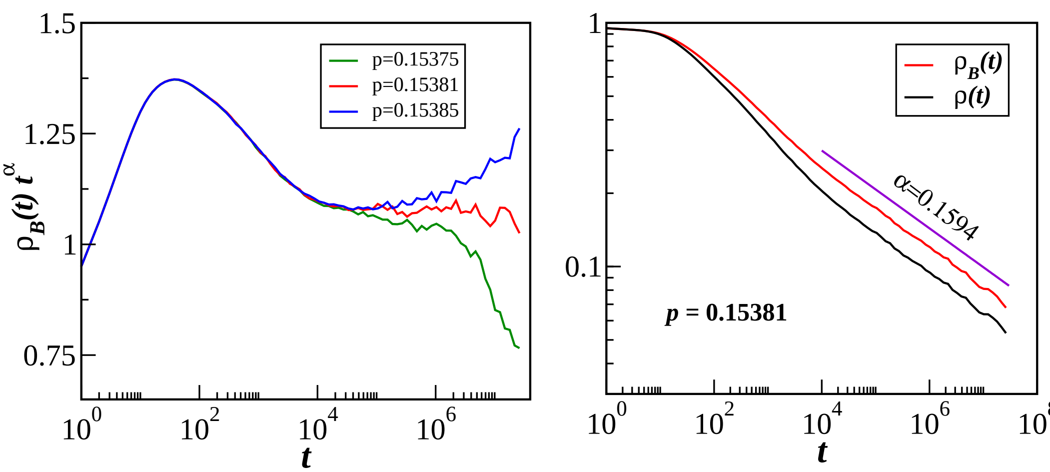


Figure 1. (a) Plots of $\rho_B(t)t^\alpha$ vs t for $p = 0.15375, 0.15381, 0.15385$ with $q = 0.02$, $r = 0.9$ and $L = 2^{20}$. Here we use DP critical exponent $\alpha = 0.1594$. (b) Both $\rho(t)$ and $\rho_B(t)$ asymptotically decay as $t^{-0.1594}$ at $p_c = 0.15381$.

To check this we redo the Monte-Carlo simulation for larger system size $L = 2^{20}$ and measured $\rho_B(t)$ up to $t = 3 \times 10^7$ MCS for different values of p, while keeping

$q = 0.02$ and $r = 0.9$. From Fig. 1(a) it is evident that $\rho_B(t)t^\alpha$ shows a saturation at $p_c = 0.15381$ for $\alpha = \alpha_{DP} = 0.1594$. Figure 1(b) shows log-scale plot of $\rho(t)$ and $\rho_B(t)$ at p_c; it is evident that after a long transient $\rho_B(t)$ decays as $t^{-\alpha}$ with $\alpha = 0.1594$, similar to $\rho(t)$. Such a change in α to a lower value was appearing as a saturation in ρ_B, leading to a lower estimate of the critical point.

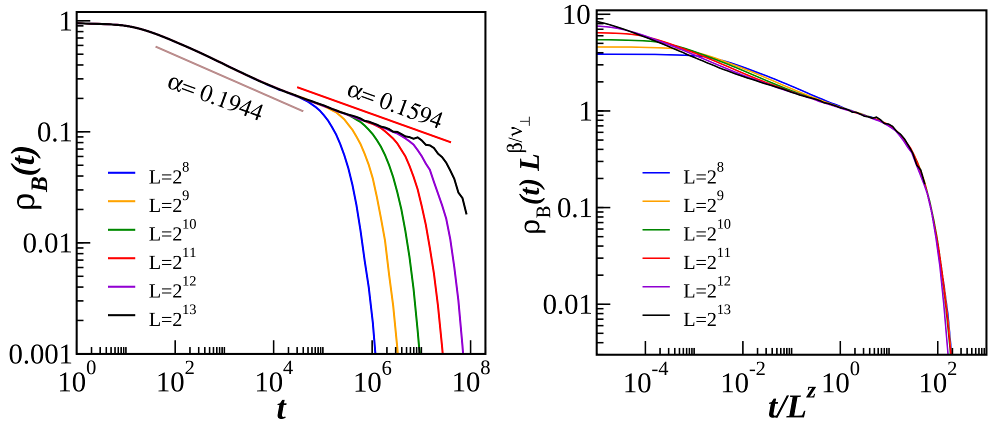


Figure 2. (a) Log-scale plots of $\rho_B(t)$ vs t at criticality ($p = 0.15381$, $q = 0.02$ and $r = 0.9$)for $L = 2^8, 2^9, 2^{10}, 2^{11}, 2^{12}$ and 2^{13}. The initial slope $\alpha = 0.194$ crosses over to $\alpha_{DP} = 0.1594$ as system size is increased. (b) The same data are collapsed according to Eq. (3) by using DP critical exponents $\beta/\nu_\perp = 0.252$, and $z = 1.580$,

With this correct estimation of critical point, we proceed to calculate other critical exponents taking ρ_B as the order parameter. As for finite system of size L, starting from a high density of predators, $\rho_{_B}(t, L)$ decays as $t^{-\alpha}$, indicating a scaling form

$$\rho_{_B}(t, L) = L^{-\beta/\nu_\perp}\tilde{\mathcal{G}}(t/L^z), \tag{3}$$

at the critical point, where z is the dynamical critical exponent. Thus, $\rho_{_B}(t)L^{\beta/\nu_\perp}$ for different values of L are expected to collapse to a single function when plotted against t/L^z. Figure 2(a) shows decay of $\rho_{_B}(t)$ for different L starting from the configuration $\rho_A = 1 = \rho_B$; clearly small systems show an effective exponent $\alpha = 0.194$ which crosses over to α_{DP} as the system size is increased. True finite size effect of the critical point sets in at a reasonably large L. The data collapse according to Eq. (3) is observed in Fig. 2(b) where we use the DP exponents $\frac{\beta}{\nu_\perp} = 0.252$ and $z = 1.580$.

In summary, although there are many absorbing configurations in 4SPP model, the numerical simulations suggests that only one of them ($\rho_A = 1, \rho_B = 0$) is dynamically accessible. The critical behavior of the absorbing transition can be well described by taking ρ_B as an order parameter. The earlier estimated values of the critical exponents vary slowly with system size and settle to DP values. These studies truly emphasize the complications, difficulties and danger associated in numerical determination of universality class, when the possibility of a long transient is not ruled out.

Acknowledgement : PKM would like to acknowledge M. Basu and U. Basu for doing independent Monte-Carlo simulations to confirm the results.

References

[1] J. Marro and R. Dickman, *Nonequilibrium phase transitions in lattice models* (Cambridge University Press, Cambridge, 1999).

[2] H. Hinrichsen, Advances in Physics **49**, 815 (2000); M. Henkel, H. Hinrichsen, and S. Lübeck, *Non-Equilibrium Phase Transitions*, Berlin: Springer (2008).

[3] P. Grassberger, Z. Phys. B **47**, 365 (1982); H. K. Janssen, Z. Phys. B **42**, 151 (1981).

[4] J. L. Cardy and R. L. Sugar, J. Phys. A: Math. Gen. **13** L423 (1980).

[5] A. Lipowski and M. Lopata, Phys. Rev. E **60**, 1516(1999). M. Basu, U. Gayen, and P. K. Mohanty, *arXiv:1102.1631*; A. Ghosh, U. Basu, A. Chakraborti, and B. K. Chakrabarti, Phys. Rev. E **83**, 061130 (2011) .

[6] I. Jensen, Phys. Rev. Lett. **70**, 1465, (1993); I. Jensen and R. Dickman, Phys. Rev. E **48**, 1710 (1993); J. F. F. Mendes, R. Dickman, M. Henkel, and M. C. Marques, J. Phys. A **27**, 3019 (1994); D. ben- Avraham and J. Kohler, J. Stat. Phys. **65**, 839 (1992); E. V. Albano, J. Phys. A **25**, 2557 (1992).

[7] P. K. Mohanty and D. Dhar, Phys. Rev. Lett. **89**, 104303 (2002); Physica A **384**, 34(2007).

[8] M. Rossi, R. Pastor-Satorras, and A. Vespignani, Phys. Rev. Lett. **85**, 1803 (2000); S. Lubeck and P. C. Heger, Phys. Rev. E **68**, 56102 (2003).

[9] T. Nagatani, Jnl. Phys. Soc. Japan **61**, 3196 (1992); J. D. Noh and H. Park, Phys. Rev. Lett. **94**, 145702 (2005); S. Kwon and Y. Kim, Phys. Rev. E **72**, 066122 (2006).

[10] S. Cornell, M. Droz, R. Dickman and M. C. Marques, J. Phys. A: Math. Gen. **24** 5605 (1991); H. K. Janssen, Phys. Rev. Lett. **78**, 2890 (1997); J. Stat. Phys.**103**, 801 (2001); U. C. Täuber, M. J. Howard, and H. Hinrichsen, Phys. Rev. Lett. **80**, 2165 (1998); J. L. Cardy and U. C. Täuber, Phys. Rev. Lett. **77**, 4780 (1996); J. Stat. Phys. **90**, 1 (1998).

[11] E. Arashiro and T. Tomé, J. Phys. A: Math. Theor. **40**, 887 (2007); A. Lipowski and D. Lipowska, Physica A **276**, 456 (2000).

[12] R. Chatterjee, P. K. Mohanty and A. Basu, J. Stat. Mech. L05001 (2011).

[13] S-C. Park, arXiv:1108.5127